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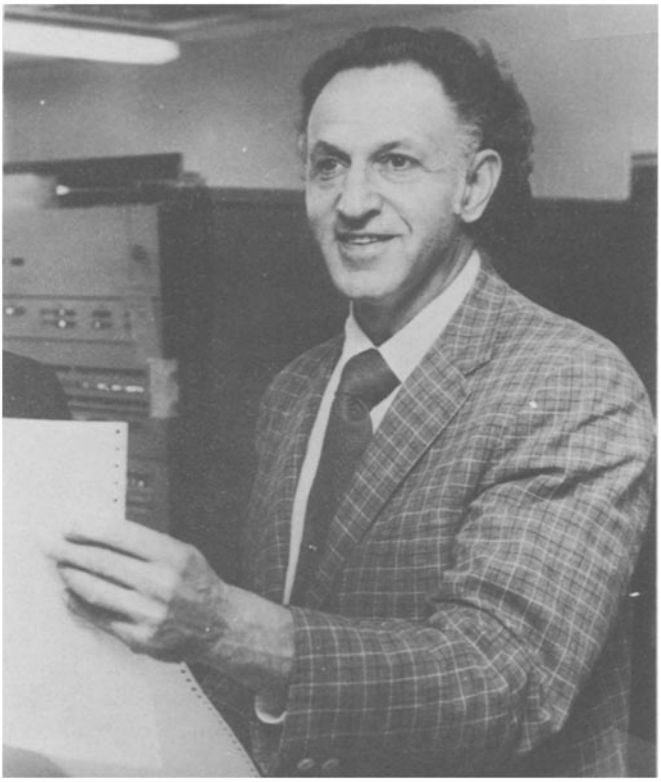
FOREWORD

An Appreciation of Professor Richard Bellman

This issue of *Journal of Optimization Theory and Applications* is dedicated to Professor Richard Bellman, a most prolific and renowned researcher in applied mathematics with a number of profoundly significant contributions to his credit. The editors of this journal honor him for his pioneering work and for what has already been, and will most surely be in the future, his long-lasting influence on applied mathematics in a remarkable variety of areas throughout the world.



Professor Richard Bellman.

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Vita. Richard Bellman was born on August 26, 1920 in New York City. He received the BA degree from Brooklyn College in 1941, the MA degree from the University of Wisconsin in 1943, and the PhD degree from Princeton University in 1946, all in mathematics. He taught in the Departments of Mathematics of Princeton University and Stanford University from 1946 to 1952. From 1953 to 1965, he worked at RAND Corporation in Santa Monica, California. In 1965, he became Professor of Mathematics, Electrical Engineering, and Medicine at the University of Southern California; he continues to hold this position today.

Professor Bellman is well established as one of the most prolific technical authors of all time. His publications include over 600 research papers in technical journals, over 30 books, and seven monographs. He has also been the Editor of two major journals, the *Journal of Mathematical Analysis and Applications* and the *Journal of Mathematical Biosciences,* both of which he was instrumental in founding. He is also the Editor of two major book series, *Mathematics in Science and Engineering* and *Computational Methods in Science and Engineering.*

His honors and awards include:

Norbert Wiener Prize, Applied Mathematics, American Mathematical Society and Society for Industrial and Applied Mathematics, 1970;

Dickson Prize, Carnegie-Mellon University, 1970;

Alza Distinguished Lectureship, Biomedical Engineering Society, 1972;

Doctor of Science, University of Aberdeen, Scotland, 1973;

Doctor of Laws, University of Southern California, 1974;

Doctor of Mathematics, University of Waterloo, Canada, 1975;

Fellow, American Academy of Arts and Sciences, 1975;

John Von Neumann Theory Prize, Institute of Management Sciences and Operations Research Society of America, 1976;

Member, National Academy of Engineering, 1977;

Medal of Honor, Institute for Electrical and Electronic Engineers, 1979.

Scientific Work. The scientific work of Professor Bellman has been extremely prolific. We present here some of the highlights.

To understand the mathematics of Richard Bellman's work, one has to understand that it is a reflection of his scientific philosophy. He believes that important mathematics comes from important scientific problems. If one works on an important scientific problem, then one is more likely to develop important results.

He started in stability theory, which is the study of systems subject to small deterministic influences. From there, it was a natural step to go to the study of systems subject to large stochastic influences. This was some of the first important work on modern control theory.

He became very interested in obtaining answers through numerical techniques. When computers became available in the early 1950's, this led to research in many directions. First of all, he wanted to know what computers could do. This led to work in both classical mathematics and artificial intelligence. He also became interested in simulation, which he applied to business games and psychiatric interviewing. Then, he went on to study what classical mathematics could not do and what computers could do. From there, he became intrigued with what neither classical mathematics nor computers could do. This led to his study of combinatorial processes, which included puzzles and scheduling. The computer led to two scientific revolutions. First, it allowed 18th century mathematics to be done very quickly. Then, it led to the development of new algorithms for the new device: 20th century technology deserves 20th century mathematics.

From the late 1940's on, Bellman became interested in decision making. This led to the theory of dynamic programming. The realization that some processes could be regarded as multistage decision processes furnished new algorithms.

Bellman is best known for his work on *dynamic programming*. During a very short time period in the 1950's, he single-handedly brought forth the theory of dynamic programming and applied it to problems in numerous fields of engineering, mathematics, and science. His book, *Dynamic Programming*, published by Princeton University Press in 1957, burst upon the scene with a detailed description of dynamic programming, a mathematically rigorous framework to support the concepts, numerous problems to which the method had been applied, and countless ideas for further research. Rarely does a single individual in a single document so completely define a major field of endeavor. Indeed, the 1957 book is still a major source of material for dynamic programming and an outstanding place to find new research ideas, almost unique for a 23-year old book in a fast-moving field.

The great power and lasting value of dynamic programming is due in large measure to its compatibility with the digital computer. In the early 1950's, Bellman was one of the first to appreciate the vast potential computing power of the then primitive device, and he sought to develop a new theory of decision making that could utilize this capability. He was led to examine multistage decision processes, in which a sequence of decisions is to be made, rather than a single decision. He then stated the *principle of optimality*; this principle, which is the cornerstone of dynamic programming, allows the sequence of decisions to be computed in a series of computations, each of which has complexity on the order of the computation for a single decision. This recursive type of calculation is ideally suited for implementation on a digital computer; thus, the staggering reductions that have taken place in the time and cost required for computation since the 1950's, as a result of both hardware improvements and new concepts in computer architecture, have increased directly the applicability of dynamic programming. Furthermore, the total computation time required grows linearly with the number of stages (i.e., the number of decisions in the sequence), rather than exponentially, as would be the case with algorithms for computing these decisions simultaneously. Thus, dynamic programming opened up a whole new class of decision-making problems to analysis and paved the way for a very substantial part of what we know now as modern control theory.

The impact on the field of control theory was immediate. At that time, the field emphasized steady-state control and frequency-domain methods, and it was filled with papers on various aspects of transient control for dynamic systems using time-domain methods. Discrete-time systems received equal attention with continuous-time systems. Many analytic results were published, such as those pertaining to the LQ regulator problem; many applications were attempted; and many additional theoretical results were obtained.

The impact of dynamic programming became even greater during the latter half of the 1960's and the early part of the 1970's, when the role of uncertainty in control systems came to be appreciated. The basic mathematical framework of dynamic programming was able to accommodate uncertainties of many different types, and the principle of optimality was extended to the stochastic case. The 1957 book, *Dynamic Programming*, contained considerable material on stochastic decision processes, and the 1961 book, *Adaptive Control Processes: A Guided Tour*, also published by Princeton University Press, presented many additional results and applications.

Dynamic programming was the key to proving the separation theorem for LQG problems, and it greatly influenced innumerable other papers on the topics of stochastic control and artificial intelligence. Bellman himself has written many articles and books on all of these topics and has made numerous theoretical and applied contributions to these fields. One of his most recent books, *Can Computers Think? An Introduction to Artificial Intelligence*, published by Boyd and Fraser Publishing Company in 1978, is an especially good summary of how his ideas apply to computer decision making in an extremely broad range of problems.

Although dynamic programming may be his best-known contribution, much of his other work has had great influence as well. In terms of specific theoretical and computational work in the control field, his early results on bang-bang control theory and his development of quasilinearization as a computational technique are good examples. Much of his fundamental work on the theory of differential equations, difference equations, and differential-difference equations has had great impact on mathematical systems theory and stability theory; his two-volume set, *Introduction to the Mathematical Theory of Control Processes*, published by Academic Press in 1968 and 1971, contains many of these results. Some of his basic work on inequalities has also been of great value to workers in the stability field. His well-known work on matrix theory has also been very influential. Another area where he did pioneering work was the application of computers to simulation, particularly simulation of control and decision processes. There are many other areas related to control theory where his work has been very important, but this list is representative.

A discussion of Bellman's contributions would not be complete without mentioning his work on applications. Bellman has always been interested in problems from all fields of human endeavor, and he has never been afraid to tackle any area that interested him, no matter how remote it might be from abstract mathematics.

Perhaps, one of his boldest moves in a new direction was his work in the medical field. Beginning in the 1960's, he recognized that many biological systems display a number of characteristics similar to those of the decision processes to which he had devoted much attention. He then turned his talents toward developing models and control policies for these systems. He has published many excellent papers in this field and has achieved recognition as one of the pioneers in bringing the strength of mathematics and computer science into the medical area. The development of the computer made it possible for the first time to construct realistic models in biology and medicine. The application of mathematics to medical problems had a strong influence on Bellman's research. His original motivation was the cancer problem; it is still a mystery why the control mechansim of a cell suddenly fails.

A problem in cardiology led to the theory of identification. The fact that organs are irregular and nonhomogeneous led to the study of two-dimensional and three-dimensional partial differential equations over irregular regions. A problem in chemotherapy required the numerical solution of a differential-difference equation. The method developed for this problem led to a way of treating successive approximations without requiring the storage of the last approximation. The side-effect problems of chemotherapy and radiotherapy could be treated by control theory. Hypodermic injections led to impulse control.

Most recently, Bellman has become interested in what might be called *selective computation*. Here, the problem is to compute what is wanted, without computing what is not wanted. He is also working actively in the study of energy, in particular, coal energy, geothermal energy, and solar energy.

At first glance, Bellman's interest in number theory seems to be at variance with the above statements. However, this is not the case. Numbers are real, and the regularity observed in tables is a challenge to the mathematician. This has been the case with many famous mathematicians, including Euler, Gauss, and Poincaré.

Bellman has always been interested in mathematical education. He has written many expository articles, and all of his books are written in such a way that they convey new ideas and new methods to students and researchers alike.

Of course, he has made major contributions in numerous areas of applications, including aerospace engineering, electrical engineering, chemical engineering, mechanical engineering, civil engineering, energy, economics, ecology, water resources, mathematical physics, operations research, management science, psychology, and sociology.

Richard Bellman is truly a modern-day Renaissance man. His breadth of interests and his ability to contribute to so many fields on so many different levels is rare indeed. It is most appropriate that the editors of this journal have chosen to recognize him by means of this testimonial volume.

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